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TURBULENT FORCED FLOW AND HEAT EXCHANGE IN VERTICAL CHANNELS IN CONDITIONS OF FREE CONVECTION

A. F. Polyakov

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Based on a unified approach, data are analyzed and generalized concerning the distributions of velocity and temperature, frictional resistance and heat transfer in the case of turbulent free-convective flow, and forced flow in conditions of the significant effect of the gravitational field.

Turbulent forced flow in vertical channels from below upwards with heating, and from above downwards with cooling, is considered under the conditions of action of the gravitational field. We call the limiting case of the very strong effect of buoyancy, the "free convection mode."

First of all, we consider free turbulent convection close to the vertical surfaces from the general positions of the boundary flows.

In the first place, we will be interested in a boundary condition of the second species, and therefore we describe the dimensionless numbers related with the thermal effect by the quantity q_C. However, heat exchange in the case of turbulent forced flow is almost no different with the boundary conditions q_C = const and t_C = const. Even the turbulent free convective flow is quite conservative during transition from the boundary condition q_C = const to t_C = const. In particular, the data of a number of papers confirm this, showing the independence of the heat-transfer coefficient $\alpha = q_C/(t_C - t_\infty)$ on the longitudinal coordinate x, i.e., for a specified constant value of q_C or t_C the other quantity correspondingly is also constant.

In Fig. 1 the temperature distribution in the case of forced turbulent boundary flow of air [1, 2] and turbulent free convective flow of air along a vertical plate [3] are compared in the universal coordinates $T^+-\eta$. In [3] the tangential stress on the wall τ_C was measured so that the friction velocity v_* is determined by the experimental data. It can be seen that the temperature distribution in universal coordinates in the case of free convection coincides with the temperature distribution in the case of forced flow without the effect of mass forces. This distribution is described by the following interpolation relation:

$$T_\tau^+ = 2.2\ln(1+0.45\mathrm{Pr}\,\eta) + (13\mathrm{Pr}^{2/3} - \ln\mathrm{Pr} - 4)\,[1 - \exp(-\mathrm{Pr}^{3/4}\eta^{1.5}/50)], \quad (1)$$

corresponding with an accuracy of ±7% to the most reliable experimental data assembled in [1, 2, 4, 5], and the results of calculations given in these papers, over the range of Pr values from 0.02 to 64.

In [6], for Pr ≅ 16, the relation

$$\frac{\tau_c}{\rho\,[\beta g\,(t_c - t_\infty)\,\nu]^{2/3}} = \mathrm{const} \quad (2)$$

is obtained, from which it follows that

$$\mathrm{Gr}_x\, T_\infty^+/x_+^4\,\mathrm{Pr} = A \approx 0.18, \quad (2a)$$

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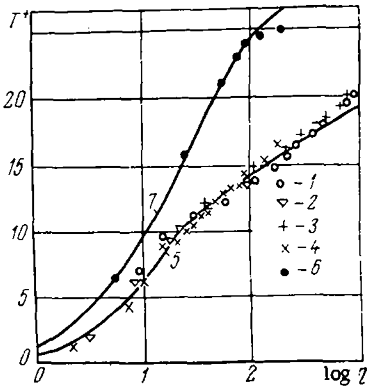

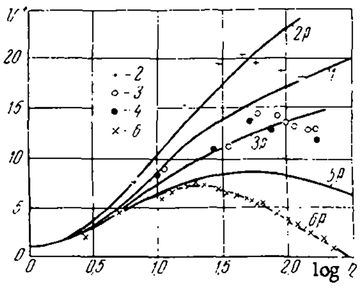

Fig. 1

Fig. 2

Fig. 1. Temperature distribution for Pr = 0.7. Experimental data: 1-3) forced flow in tubes, Gr → 0, Re = $7.1 \cdot 10^4$ [1], Re = $2.5 \cdot 10^4$ [1], Re = $1.8 \cdot 10^5$ [2], respectively; 4) free convective streamline flow with a vertical surface [3], $Gr_x = 2 \cdot 10^{12}$; 6) forced flow, Re = $9.8 \cdot 10^3$, E = $1.9 \cdot 10^{-8}$ [15]. Calculation: 5) by Eq. (1), Gr → 0; 7) by Eq. (30), Re = 10^4, E = $1.9 \cdot 10^{-8}$.

Fig. 2. Velocity distribution: 1) Gr = 0, calculation by Eq. (7a). Experimental data: 2-4) forced flow in tubes, Re = $5 \cdot 10^3$, Pr = 0.7; E = $5.1 \cdot 10^{-8}$ [17], $1.8 \cdot 10^{-7}$ [16], $1.8 \cdot 10^{-7}$ [15], respectively; 6P) free convective streamline flow with a vertical surface [3], $Gr_x = 2 \cdot 10^{12}$. Calculation: 2P, 3P) by relation (7b) for the parameters corresponding to the experimental data 2, 3; 5P) by Eq. (7b) for E = 10^6; 6P) by Eq. (7).

where

$$T^+_\infty = 2.2\ln\eta_0 + 12.7\mathrm{Pr}^{2/3} - 6, \tag{1a}$$

is obtained from Eq. (1) for large values of η and Pr > 0.6. From the data of [3] for air (Pr = 0.7), it follows that A ≅ 0.07.

Thus, A is a function of the Pr number, which on the basis of these data and data on the heat transfer can be determined as:

$$\mathrm{Gr}_x T^+_\infty / x^4_+ \mathrm{Pr} = 0.1\,\mathrm{Pr}^{1/3}. \tag{3}$$

The value of the dimensionless temperature difference between the wall and liquid at a distance from the wall T^+_∞ is related with the Nusselt number by the expression

$$\mathrm{Nu}_x = \frac{q_c x}{(t_c - t_\infty)\lambda} = \frac{\mathrm{Pr}\, x_+}{T^+_\infty}. \tag{4}$$

On the basis of the data given in [3, 7], the outer boundary of the free-convective turbulent boundary layer η_0 can be described by the relation

$$\eta_0 = 0.16\,\mathrm{Gr}_{\Delta t}^{1/3} = 0.28\,\mathrm{Gr}_x^{1/4}\,\mathrm{Pr}^{1/12}. \tag{5}$$

From Eqs. (3) and (4) for the Nu_x number we obtain

$$\mathrm{Nu}_x = 1.8\,\mathrm{Pr}^{2/3}\,\mathrm{Gr}_x^{1/4} / (T^+_0)^{3/4}. \tag{6}$$

The calculation of the Nu_x number by relations (6), (2a), and (5) coincides well with the experimental data of [3, 6, 8], obtained over the range of Pr values from 0.7 to 16. In this case, the physical properties are chosen for a temperature equal to the arithmetic mean value of the temperatures of the wall and at a distance from the surface.

Using the results of [3, 7], the velocity distribution can be approximated by the following relation:

$$U^+ = U^+_T + 2.5Y(\ln Y - 2) - (2.5\ln\eta_0 - 0.5)\begin{Bmatrix}(1 - 0.23\ln Y) & \text{for } Y > 0.015\\ (12Y)^2 & \text{for } Y < 0.015\end{Bmatrix}, \tag{7}$$

where the velocity distribution U_T^+ for forced flow in universal coordinates is described by a relation which does not take account of a certain additional effect of the Re number close to the axis:

$$U_T^+ = 2.5\ln(1 + 0.4\eta) + 7.8\,[1 - \exp(-\eta^{1.7}/47)]. \tag{7a}$$

The velocity profile, calculated by interpolation equation (7), is compared in Fig. 2 with the experimental data of [3].

Relation (3), connecting the dynamic characteristics of the boundary free-convective turbulent flow, can be written not in terms of the longitudinal coordinate x (parameters Gr_x and x_+), but in terms of the thickness of the boundary layer δ (parameters Gr and η_0). From this relation, for the case of flow in a channel in the free convective mode, we determine the dimensionless radius $\eta_0 = v_* d/2\nu$:

$$\eta_0 = (T_0^+\,\mathrm{Gr}/1.6\,\mathrm{Pr}^{4/3})^{1/4}. \tag{8}$$

First of all we find the relation for the heat transfer and the frictional resistance in circular tubes, in the free convection mode. Then, we generalize them and the data obtained earlier for the small effect of buoyancy [9], on the case of change of the defining parameters from forced flow conditions to the free convection mode.

The free convection mode in the case of forced flow in vertical channels will be defined as the conditions with combination of the direction of forced flow and the thermal boundary conditions, contributing to the development of free-convective flow (e.g., flow from below upwards in heated tubes), when relation (8), which is characteristic for free-convective turbulent flow, is satisfied.

Let us consider the flow at a distance from the origin of heating (cooling) for x/d > 40, when the heat-exchange conditions for turbulent flow can be assumed to be stabilized.

As shown above, the distribution of the dimensionless temperature T^+ for free convection and for forced flow is described by one and the same expression (1). Consequently, there is every basis for supposing that this same expression will describe the temperature distribution also in the free convection mode in the case of flow in a channel.

We find the velocity distribution in the free convection mode by the relation, similar to Eq. (7):

$$U^+ = U_T^+ + b\,[2.5Y(\ln Y - 2) - (2.5\ln\eta_0 + 0.5)(1 + 0.23\ln Y)]. \tag{7b}$$

By comparison with Eq. (7), we introduce the coefficient b into Eq. (7b), equal to zero for forced turbulent flow and with a certain limiting value b_{fc} in the free convection mode.

We determine the Stanton number by the temperature and velocity distributions we have found, from the equation

$$\frac{1}{\mathrm{St}} = 2\int_0^1 U^+T^+(1 - Y)\,dY. \tag{9}$$

For integration, we use the simplified expression for the temperature distribution

$$T_T^+ = 2.2\ln\eta + 12.7\,\mathrm{Pr}^{2/3} - 6 \tag{1b}$$

and in the velocity distribution (7b), we use the simplified expression for

$$U_T^+ = 2.5\ln\eta + 5.5. \tag{7c}$$

Substituting Eqs. (1b), (7b), and (7c) in Eq. (9), integrating, and carrying out transformations and permissible simplifications, we obtain

$$1/\mathrm{St} = 5.5\ln^2\eta_0 + 31.8\,(\mathrm{Pr}^{2/3} - 0.5)\ln\eta_0 - b\,[3.6\ln^2\eta_0 + 20.3\,(\mathrm{Pr}^{2/3} - 0.37)(\ln\eta_0 + 1.7) - 9.4]. \tag{10}$$

For b = 0, expression (10) describes the heat transfer for forced flow, without the effect of mass forces St_T, and for $b = b_{fc}$ – the heat transfer in the free convection mode. The results of the calculation by Eq. (10) for Pr > 0.6 and b = 0 and correspond well with the results of the calculation by the equation [10]

$$\mathrm{St}_T = \frac{\xi/8}{1 + \dfrac{900}{\mathrm{Re}} + 12.7\sqrt{\xi/8}\,(\mathrm{Pr}^{2/3} - 1)}. \tag{11}$$

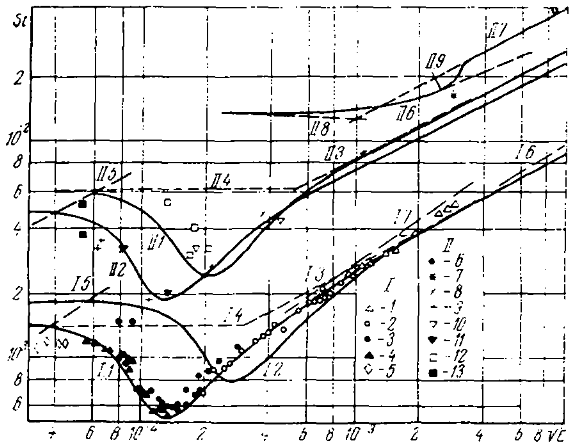

Fig. 3. Dependence of the St number on E. I: Pr number of 6: 1,2,3,4,5,) experimental data [18] for Re = (1.8–2.6)·10^3, (3–5.2)·10^3, (6–8)·10^3, (8.5–10.5)·10^3, and (1.2–1.5)·10^4, respectively; I.1) calculated according to (33) for Re = 10^4; I.2) calculated according to (33) for Re = 5·10^3; I.3) calculated according to (23); I.4) calculated according to (11) for Re = 10^4; I.5) calculation of the boundary of the initial effect of buoyancy according to the dependence given in [9]; I.6) calculation according to the interpolation dependence obtained in [19] for Re = 300; I.7) calculation according to [19] for Re = 2·10^3. II: Pr number of 0.7: 6, 7, 8, 9) experimental data of [16] for Re = 500, (1–2)·10^3, (4–5)·10^3, (0.9–1.3)·10^4; 10, 11) experimental data of [15] for Re = (5–7)·10^3 and 10^4, respectively; 12, 13) experimental data of [17] for Re = 5·10^3 and 10^4, respectively; II.1) calculation according to (33) for Re = 5·10^3; II.2) calculation according to (33) for Re = 10^4; II.3) calculation according to (23); II.4) calculation according to (11) for Re = 5·10^3; II.5) calculation of the boundary of the initial effect of buoyancy according to the dependence given in [9]; II.6) calculation according to the dependence for the viscous-gravitational regime [14], Re = 500; II.7) calculation according to (23); II.8) calculation according to the dependence for laminar flow, Nu = 4.36, X > 0.07; II.9) boundary of the transition from viscous-gravitational to turbulent flow, calculated according to the dependence obtained in [14].

In this case, the coefficient of frictional resistance ξ is calculated by one and the same relation, e.g., by Filonenko's equation [11],

$$\xi_T = \left[1.82\lg\left(\frac{\text{Re}}{8}\right)\right]^{-2}. \tag{12}$$

From the equation

$$\bar{U}^+ = \sqrt{2/c_f} = \sqrt{8/\xi} = 2\int_0^1 U^+ R\,dR,$$

using Eqs. (7b) and (7c), we determine the coefficient of frictional resistance ξ:

$$\sqrt{8/\xi} = 2.5\,(\ln 2\eta_0) - 1.65b\,(\ln 5.2\eta_0). \tag{13}$$

Expressing η_0 in terms of the coefficient of frictional resistance

$$\eta_0 = \frac{\text{Re}}{2}\sqrt{\frac{c_f}{2}} = \frac{\text{Re}}{2}\sqrt{\frac{\xi}{8}}, \tag{14}$$

we write Eq. (13) in the form

$$\sqrt{8/\xi} - 2.5\,(1 - 0.66b)\ln\sqrt{\xi/8} = 2.5\,(1 - 0.66b)\ln(\text{Re}/2) - 1.8\,(1 - 1.5b). \tag{15}$$

When b = 0, Eq. (15) represents the relation for the coefficient of frictional resistance, obtained by L. Prandtl (see, e.g., [12]), which describes well the experimental data in the range 2.8·10^3 < Re < 10^7.

In the range of variation of ξ from 10^{-3} to 0.3, the results of the calculations by Eq. (15) can be approximated by the expression

$$\xi = \left[1.82\,(1 - 0.66b)\left(\lg\frac{\text{Re}}{8}\right)\right]^{-2}, \tag{16}$$

which, for b = 0, converts to Eq. (12).

Expressing the quantity $\ln\eta_0$ from Eq. (13) and substituting it in Eq. (10), we find the relation between the St number and the coefficient ξ:

$$\mathrm{St} = \frac{\xi/8}{\dfrac{0.88}{1-0.66b} + \sqrt{\dfrac{\xi}{8}}\left[12.7\,(\mathrm{Pr}^{2/3} - 0.88) + \dfrac{1.2b}{1-0.66b}\right]}. \tag{17}$$

Values of the St number, calculated by Eq. (17) for b = 0 and Pr > 0.6, agree well with the values determined by Eq. (11). Bearing this in mind, we represent Eq. (17) in the form corresponding to the limiting transition for b = 0 to the well-known previous relation (11), i.e.,

$$\mathrm{St} = \frac{\xi/8}{\dfrac{1+\mathrm{Re}/900}{1-0{,}66b} + 12.7\sqrt{\dfrac{\xi}{8}}\left(\mathrm{Pr}^{2/3} - 1 + \dfrac{0.1b}{1-0.66b}\right)}. \tag{18}$$

The coefficient of frictional resistance in the free convection mode can be determined from Eqs. (8) and (12):

$$\frac{\xi_{fc}}{8} = 3.2\sqrt{\frac{T_0^+}{\mathrm{Pr}^{1/3}}\,\mathrm{E}}, \tag{19}$$

where $\mathrm{E} = \mathrm{Gr}/\mathrm{Pr}\cdot\mathrm{Re}^4$.

In the range $0.6 < \mathrm{Pr} < 100$ and $100 < \mathrm{Re} < 10^5$, calculations by Eq. (19) can be closely approximated in the form

$$\xi_{fc} = 120\sqrt{\mathrm{E}}. \tag{20}$$

We express the quantity b from Eq. (16):

$$b = 1.52\left(1 - \frac{0.55}{\sqrt{\xi}\,\lg 0.125\,\mathrm{Re}}\right). \tag{21}$$

Substituting the value of ξ_{fc} from Eq. (20) in Eq. (21), we obtain the expression for b in the free convection mode:

$$b_{fc} = 1.52\left(1 - \frac{0.05}{\mathrm{E}^{1/4}\,\lg(\mathrm{Re}/8)}\right). \tag{22}$$

Substituting in Eq. (18) expression (20) and (22), we obtain the relation for the St_{fc} number in the free convection mode:

$$\mathrm{St}_{fc} = \frac{\mathrm{E}^{1/4}}{1.33\,(\lg \mathrm{Re}/8) + 3.3\,(\mathrm{Pr}^{2/3} - 0.7)}. \tag{23}$$

Figure 3 shows the change of St as a function of the parameter E, for values of Pr = 0.7 and Pr = 6. In the case of Pr numbers ≅1, a marked additional dependence on the Re number is observed, in addition to that taken into account in the quantity E. For Pr > 5, the additional dependence on the Re number is insignificant. For the values of Pr = 6 shown in the graph, the curves for Re = 500 and Re = 10^4 are very little different and therefore only one line is shown. With relatively large values of E, a satisfactory agreement with the experimental data can be seen. With reduction of E, the lines calculated by Eq. (23) are above the experimental data. This is because with a relatively small degree of effect of the gravitational field on the turbulent forced flow in vertical boundary flows, as was shown in [9], the buoyancy forces in the first place affect the turbulence, which in the case being considered leads to reduction of the turbulent transfer. However, with increase of E, the effect of buoyancy on the turbulence is weakened and its effects starts to appear more and more directly on the averaged flow, leading to the free convection mode. Thus, if there were no buoyancy effect on the turbulence, then the St number would vary according to the relations corresponding to the dashed lines. If, further, the Re number corresponds to the laminar flow mode, in conditions where the gravitational field effect is absent, then the effect of buoyancy will lead to the development of a viscous-gravitational flow mechanism, which after the breakdown of stability will convert to the turbulent free convection mode. This situation is shown in Fig. 3 for the case Pr = 0.7, Re = 500, and X = (1/Pe)(x/d) > 0.07. It is shown in [13] that transition in this case from viscous-gravitational flow to turbulent flow takes place without a sharp change of heat transfer. The boundary of the transition can be determined by the relation given in [14].

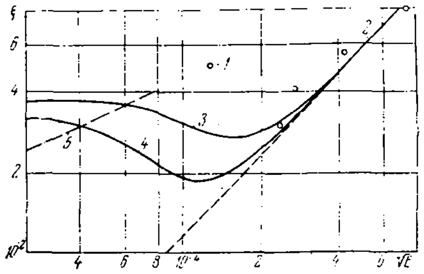

Fig. 4. Graph of the coefficient of frictional resistance vs the parameter E: 1) experimental data of [16] for Pr = 0.7 and Re = (3.6–7)$\cdot 10^2$; 2) calculation by Eq. (20); 3, 4) calculation by Eq. (28) for Re = $5 \cdot 10^3$ and 10^4; 5) limit of start of buoyancy effect by Eq. (26).

As shown in [9], as a result of the action of thermogravitational forces on turbulent transfer, the values of the velocity, represented in universal coordinates, are increased in the central part of the flow. As a result of this, the St number is decreased. If, in the transition region from the forced turbulent flow mode to the turbulent free convection mode, we use expression (7c) for describing the velocity profile, then, by selecting values of the parameter b in an appropriate way, the change of the velocity profile due to the effect of buoyancy on the turbulent transfer can be taken into account approximately.

We represent the relation for the parameter b in the form of a product

$$b = m b_{fc}, \tag{24}$$

where m = 1 in the free convection mode and m = 0 for forced flow, without the effect of buoyancy.

By using expressions (24), (22), and (16), we obtain a relation for the frictional resistance in the case of turbulent buoyant (descending) flow in vertical heated (cooled) tubes in the case x/d > 40:

$$\xi = \frac{\sqrt{E}}{3.3\left[(1 - m)\,E^{1/4}\left(\lg \frac{Re}{8}\right) + 0.05m\right]^2}. \tag{25}$$

With m = 0, relation (25) converts to Eq. (12), and with m = 1 it converts to Eq. (20).

In [9] the limits of the start of the effect of thermogravitational forces on the frictional resistance in the case of forced turbulent flow in vertical tubes are determined. We write the relation for the limiting Grashof number obtained in it, in the form

$$E_1 = 2 \cdot 10^4 / Re^{1.25}, \tag{26}$$

where E_1 corresponds to the limiting value of the parameter E = Gr/Pr$\cdot$Re^4, for which thermogravitation starts to noticeably (more than 1%) affect the frictional resistance. When E < E_1, this effect is negligibly small and m = 0.

Using the limiting relations being considered for ξ and the velocity distribution [15-17], the relation

$$m = (E/E_1)^2/[(E/E_1)^2 + 30] \tag{27}$$

is obtained for the parameter m.

Using expressions (26) and (27), we obtain from Eq. (25) the relation for the coefficient of frictional resistance valid for Pr > 0.6 and Re > $3 \cdot 10^3$, over the whole range of determining parameters, from the forced flow mode to the convection mode,

$$\xi = \left[\frac{1 + 8.3 \cdot 10^5 Re^{2.5} E^2}{1.82\left(\lg \frac{Re}{8}\right) + 7.6 \cdot 10^4 Re^{2.5} E^{7/4}}\right]^2. \tag{28}$$

Figure 4 shows the variation of the coefficient ξ as a function of the parameter E, calculated for two values of $Re = 5 \cdot 10^3$ and 10^4. The agreement with the experimental data of [16], obtained for the free convection mode, is satisfactory. The quantity ξ for $E < E_1$ corresponds to that calculated by Filonenko's equation [11]. In the region of transition from forced flow conditions to free convection conditions, $\xi < \xi_T$ and $\xi < \xi_{fc}$, and they depend significantly on both the Re number and on E. Values of the coefficient of frictional resistance in the transition region are less than in the limiting cases, due to the reduction of turbulent transfer of momentum as a result of the effect in this region of buoyancy on the turbulence.

Relation (7b), taking account of Eqs. (7a), (22), (24), (26), and (27), describes approximately the velocity distribution over the whole range from "purely" forced flow to the free convection mode. Figure 2 shows the velocity distributions for $Re = 5 \cdot 10^3$, calculated by these relations. The curve 6P corresponds to the free convection mode. The line 2P corresponds to the conditions of the predominant effect of buoyancy on turbulent transfer. The calculated curves 2P and 3P coincide with the experimental data of [15-17]. The experimental data indicate a more complex variation of the velocity profile by comparison with the calculated curve. The maxima on the velocity profiles occur considerably earlier than the calculation by relation (7b) gives. The deviation of the calculated values from the experimental values at certain points in the core of the flow reaches 25%. However, the design relation (7b) correctly shows the tendency of the velocity profile variation by comparison with the case of isothermal flow (curve 1) as a function of the effect of buoyancy on the turbulent transfer and directly on the averaged flow.

In the case of the action of the gravitational field on the turbulent transfer, the temperature profile in the case of a low degree of the buoyancy effect is described by the relation obtained in [9]

$$T^+ = T_T^+ + \frac{10\,Gr}{Pr\,Re_*^4}\,\eta, \tag{29}$$

where T_T^+ is the dimensionless temperature in the case of "purely" forced flow, described by Eq. (1).

In order to find the St number by Eq. (9), together with Eqs. (7b) and (7c) for the velocity profile, we use a relation represented in the form

$$T^+ = 2.2 \ln \eta + 12.7\,Pr^{2/3} - 6 + \beta Y^n (2 - Y) \tag{30}$$

for the temperature profile, where the value $n = 2/3$ is selected on the basis of the experimental data for the temperature distribution and heat transfer, and the parameter $\beta = 0$ in both the free convection mode and for forced flow without the effect of buoyancy.

The expression for β is determined on the basis of experimental data for the temperature and heat-transfer distribution in [15-18], and also the relations obtained in [9], describing the limits of the effect of buoyancy on the heat transfer. Thus,

$$\beta = \frac{1.1 \cdot 10^{10}\,Pr^{2/3}\,(Re^{1.25}E)^3\,(1 + 8.8\sqrt{Re^{1.25}E})}{1 + 4.3 \cdot 10^{11}\,(Re^{1.25}E)^4}. \tag{31}$$

Figure 1 shows the temperature distribution in universal coordinates with $Pr = 0.7$ for the forced flow mode without buoyancy effect, and the free convection mode also in conditions of transition from the first to the second mode, calculated by the relation

$$T^+ = T_T^+ + \beta Y^{2/3} (2 - Y), \tag{32}$$

where T_T^+ is calculated by Eq. (1) and β is calculated by Eq. (31). In transition conditions, the curves corresponding to the temperature distributions, e.g., 7, lie higher by comparison with curve 5, described by relation (1). This is due to a reduction of the turbulent heat transfer.

Now we write the final expression for the heat-transfer calculation, valid for $Pr > 0.6$ and $Re > 3 \cdot 10^3$, over the whole range of variation of the determining parameters from the forced flow mode to the free convection mode:

$$St = \frac{\xi}{8} \left\{ \frac{1 + 8.3 \cdot 10^5\,(Re^{1.25}E)^2}{1 + 4.2 \cdot 10^4\,Re^{2.5}\,E^{7/4} / \lg(0.125\,Re)} + 12.7 \sqrt{\frac{\xi}{8}} \times \right.$$
$$\left. \times \left[Pr^{2/3} \left(1 + \frac{7.2 \cdot 10^8\,(Re^{1.25}E)^3\,(1 + 8.8\sqrt{Re^{1.25}E})}{1 + 4.3 \cdot 10^{11}\,(Re^{1.25}E)^4} \right) - \frac{1 - 5.8 \cdot 10^5\,(Re^{1.25}E)^2}{1 + 8.3 \cdot 10^5\,(Re^{1.25}E)^2} \right] \right\}^{-1}. \tag{33}$$

With large values of E (on the order of 10^{-6} or more), relation (33) converts to the free convection mode relation (23), and with small values of E (on the order of 10^{-9} or less) it converts to relation (11) for forced flow without the buoyancy effect.

In Fig. 3 the results of a calculation by Eq. (33) are compared with the experimental data of [15-18]. The experimental data for Pr = 2-6, given in [19] in other coordinates, unfortunately cannot be replotted on Fig. 3. The calculation by the interpolation relation proposed in [19] (and approximating piecewise in St−E coordinates the experimental data of [19]) is shown in Fig. 3 for Pr = 6. The agreement between these data and the calculation by Eq. (33) may be noted.

NOTATION

c_p	is the specific heat;
d	is the diameter of cylindrical channel;
q	is the thermal flux density;
t	is the temperature;
u	is the velocity;
$v_* = \sqrt{\tau_c/\rho}$	is the frictional velocity;
x	is the longitudinal coordinate;
y	is the coordinate normal to the wall;
β	is the coefficient of volume expansion;
δ	is the thickness of boundary layer;
ν	is the kinematic viscosity;
ρ	is the density;
ξ	is the coefficient of frictional resistance;
τ	is the tangential stress;
$Gr_{\Delta t} = g\beta x^3(t_c - t_\infty)/\nu^2$, $Gr_x = g\beta x^4 q_c/\nu^2\lambda$, $Gr = g\beta d^4 q_c/\nu^2\lambda$	are the Grashof number;
$E = g\nu\beta q_c/\rho c_p \bar{u}^4 = Gr/Pr\,Re^4$	is a dimensionless number;
$Nu_x = q_c x/(t_c - t_\infty)\lambda$, $Nu = q_c d/(t_c - t_{mmt})\lambda$	is the Nusselt number;
Pr	is the Prandtl number;
$Re = \bar{u}d/\nu$, $Re_* = v_* d/\nu$	are the Reynolds number;
$St = Nu/Pr\,Re$,	is the Stanton number;
$T^+ = [(t_c - t)/q_c]\rho c_p v_*$	is the dimensionless temperature;
$U^+ = u/v_*$	is the dimensionless velocity;
$X = (1/Pr\,Re)(x/d)$	is the scaled length;
$\eta = (v_*/\nu)y$	is the dimensionless coordinate;

Subscripts

mmt	is the mean mass temperature;
c	is the value at the wall;
fc	is the case of free convection;
T	is the case of turbulent forced flow without buoyancy effect;
0	is the value on the axis;
∞	is the unperturbed flow;

a line above a symbol denotes the average over the flow cross section.

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FORCED CONVECTION IN A PLANE CHANNEL WITH RECESSES

Yu. A. Gavrilov, G. N. Dul'nev, and A. V. Sharkov

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An approximate method is suggested for estimating the coefficient of convective heat exchange in fluid flow in a flat channel with rectangular recesses in the walls.

We will consider convective heat exchange in steady, formed, fluid flow in a flat channel with rectangular recesses in the walls (Fig. 1). These recesses are identical, are equally spaced apart, and have dimensions B and H commensurate with the width h of the channel. The convective heat-exchange coefficients are to be determined.

We could not find the solution to such a problem in the literature, although much attention is paid to the effect of roughness on heat exchange. Sandy roughness has been investigated in relatively great detail [1]. There are reports where roughness differing from the sandy kind is considered. For example, vortex flow in small recesses is studied in [2]. Their dimensions are small in comparison with the channel width, and they have almost no effect on the character of the main fluid flow. A coarser roughness in the form of a spiral protuberance of Nichrome wire on the inner surface of a round pipe is adopted in [3]. The height of the protuberance is about one-tenth the pipe diameter. The experimentally derived dependence for calculating the convective heat-exchange coefficient is valid for the particular case. A set of empirical equations for concrete forms of surfaces in heat exchangers is given in [4, 5]. Data of an experimental investigation of convective heat exchange in laminar and turbulent air flow in a channel with the geometrical parameters B = 13 mm, D = 15 mm, H = 5 mm, and h = 1-10 mm are presented in [6]. In addition to transverse recesses, the walls also had longitudinal recesses. The experimental results for these channels were generalized in the form of criterial dependence $\overline{Nu} = f(Re)$.

The longitudinal flow of a stream over a surface with a single recess or protuberance is analyzed in a number of reports, such as [7-11]. A simplified flow model is chosen in this case and one or another approximate solution of the problem is given in accordance with the adopted assumptions. In [12] the flow model is extended to a surface with protuberances arranged in a series.

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